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Effect of magnetic field on onset of Marangoni convection

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Abstract—A theoretical study is made on the onset of the Marangoni convection in the horizontal layer of an electrically conducting liquid, to which a vertical temperature gradient and a magnetic field are applied. The analytical solution is obtained for the critical condition of the onset of the Marangoni convection in an infinite liquid layer, and the numerical analysis is carried out for a finite liquid layer confined in a circular cylindrical container. The effects of the magnetic field, the Biot number at the free surface and the aspect ratio of the liquid layer are made clear. The asymptotic behavior of the critical Marangoni number for the large Hartmann number is also obtained. It is found that both the critical Marangoni number and the number of roll cells which generate at a marginal state increases with the intensity of the magnetic field, and that the effect of the aspect ratio of the liquid layer of both the critical Marangoni number and the velocity and temperature field becomes small as the magnetic field is intensified. It also becomes clear that the rolls are generated when the magnetic field is inclined, while the Bénard-type cells are generated under vertical magnetic field in the case of an infinite liquid layer.

INTRODUCTION

NATURAL convection driven by the gradient of an interfacial or surface tension due to a non-uniform temperature distribution is called thermocapillary or Marangoni convection. Such a convective flow gives rise to serious problems in several cases, for example, in crystal growth from a melt. It is often requested to suppress the onset of convection. Since buoyancy driven convection is reduced under microgravity condition, Marangoni convection, in particular, is supposed to become more important and have a decisive effect on crystal growth in space, which has been studied actively in recent years [1]. When the temperature gradient is imposed vertically on the horizontal liquid layer the top surface of which is free and cooled and the bottom is on a heated rigid wall, the Marangoni convection occurs under a certain critical condition. Such an instability problem in an infinite liquid layer was first analyzed by Pearson[2] and Nield [3] who investigated the effect of buoyancy on the onset of Marangoni convection.

When a magnetic field is imposed on an electrically conducting liquid, the liquid motion is suppressed because of the interaction between the induced electric current and the external magnetic field [4]. The magnetic field, therefore, is considered to be an effective means for suppressing the onset of convective motion so far as the liquid is electrically conductive.

Convective instability induced by buoyancy in a magnetic field has been studied by Chandrasekhar [5] and the instability problem of Marangoni convection in a magnetic field has been discussed by Nield [6] and Rudraiah *et al.* [7] for the infinite liquid layer.

However, the instability problem in a finite liquid layer confined in a container may become more important in practical cases, which makes the problem more complicated.

The onset of buoyancy convection in a box, the top and bottom surfaces of which are rigid, has been analyzed by Davis [8] and developed by Catton [9] who has used the complete set of trial functions. The onset of Marangoni convection and of buoyancy convection in a circular cylindrical container has been analyzed in a sophisticated way by Vrentas *et al.* [10], though the combined case has not been considered. These authors, however, have not investigated the effect of a magnetic field.

The objective of the present study is to make clear the effects of the magnetic field and the aspect ratio of the liquid layer on the onset of pure Marangoni convection.

A theoretical study is carried out on the instability problem of pure Marangoni convection in a horizontal layer of an electrically conducting liquid. The analytical solution is obtained for the case of the infinite liquid layer, and the numerical analysis is carried out for the finite liquid layer.

The effects of the Hartmann number, the orien-

NOMENCLATURE

A	aspect ratio, R_0/L
Bi	Biot number, equation (16)
$\mathbf{h}$	magnetic field
h_0	intensity of initial magnetic field
$\mathbf{H}$	non-dimensional magnetic field, $\mathbf{h}/h_0$
H_z	Z-component of non-dimensional magnetic field
k	non-dimensional wave number
L	depth of liquid layer
M	Hartmann number, equation (9)
Ma	Marangoni number, equation (17)
Ma_c	critical Marangoni number
Pr_m	defined by equation (7)
Q	defined by equation (6)
R_0	radius of container
T	temperature
T_w	temperature at bottom wall
T_s	temperature at free surface
$\mathbf{v}$	velocity
$\mathbf{V}$	non-dimensional velocity, $\mathbf{v}/(\kappa/L)$
V_z	Z-component of non-dimensional velocity.

Greek symbols

α	heat transfer coefficient at the free surface
η	dynamic viscosity
θ	non-dimensional temperature
κ	thermal diffusivity
μ	permeability
ν	kinematic viscosity
ρ_0	density
σ	electrical conductivity
σ_t	temperature coefficient of surface tension.

tation of the magnetic field, the Biot number at the free surface, and the aspect ratio of the liquid layer are discussed.

GOVERNING EQUATIONS

The onset of convective instability in a horizontal liquid layer as shown in Fig. 1 is considered. The perturbation equations (1)–(5) are obtained from the magnetohydrodynamic equations which are derived on the basis of the following assumptions.

(a) Liquid is incompressible and the Boussinesq approximation is valid.

(b) Magnetic permeability and other physical properties of liquid are constant.

(c) Liquid is Newtonian.

(d) Electromagnetic field obeys the Maxwell equations.

(e) Magnetic field is uniform and the angle of inclination from the vertical is denoted as δ (Fig. 1).

(f) Instability occurs as a steady convection.

Continuity equation

$$\operatorname{div}\mathbf{V} = 0. \quad (1)$$

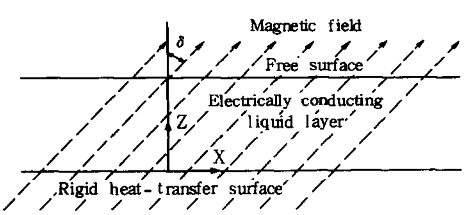

FIG. 1. Horizontal layer of an electrically conducting liquid under magnetic field.

Momentum equation

$$\Delta^2 V_z + Q\{\cos\delta\,(\partial/\partial Z) + \sin\delta(\partial/\partial X)\}\Delta H_z = 0. \quad (2)$$

Energy equation

$$V_z + \Delta\theta = 0. \quad (3)$$

Magnetic field equation

$$\operatorname{div}\mathbf{H} = 0 \quad (4)$$

$$\{(\cos\delta(\partial/\partial Z) + \sin\delta(\partial/\partial X))\}V_z + Pr_m\Delta H_z = 0 \quad (5)$$

where the coordinates, the velocity, the temperature and the magnetic field are nondimensionalized by reference quantities L, κ/L, $|T_w - T_s|$ and h_0, respectively. Parameters Q and Pr_m are defined as

$$Q = (\mu L^2 h_0^2)/(\rho_0 \kappa \nu) \quad (6)$$

$$Pr_m = 1/(\sigma\mu\kappa). \quad (7)$$

The momentum equation can be transformed into a more convenient form by eliminating H_z from equations (2) and (5)

$$\Delta^2 V_z - M^2\{\cos\delta(\partial/\partial Z) + \sin\delta(\partial/\partial X)\}^2 V_z = 0 \quad (8)$$

where M is the Hartmann number defined below which represents the intensity of the magnetic field relative to the viscous effect

$$M^2 = Q/Pr_m = (\mu^2\sigma L^2 h_0^2)/(\rho_0\nu). \quad (9)$$

Equations (1), (3), (4) and (8) are the required perturbation equations.

ONSET OF MARANGONI CONVECTION IN THE INFINITE LIQUID LAYER

Analysis of normal modes

The analysis can be made in terms of two-dimensional periodic waves for the case of an infinite liquid layer as analyzed by Chandrasekhar [5] and Pearson [2].

The perturbed variables V_z and θ can be expressed as

$$\begin{pmatrix} V_z \\ \theta \end{pmatrix} = \begin{pmatrix} F(Z) \\ G(Z) \end{pmatrix} \exp[\mathrm{i}(k_x X + k_y Y)] \quad (10)$$

where $k = \sqrt{(k_x^2 + k_y^2)}$ is the wave number of the disturbance and i represents the imaginary unit.

The following equations are derived by substituting equation (10) into equations (8) and (3):

$$[(\mathrm{D}^2 - k^2)^2 - M^2(\cos\delta \mathrm{D} + \mathrm{i}k_x \sin\delta)^2]F(Z) = 0 \quad (11)$$

$$F(Z) + (\mathrm{D}^2 - k^2)G(Z) = 0 \quad (12)$$

where $\mathrm{D}^n = \mathrm{d}^n/(\mathrm{d}z^n)$.

The equation for $G(Z)$ is obtained by eliminating $F(Z)$ from equations (11) and (12)

$$[(\mathrm{D}^2 - k^2)^3 - M^2(\cos\delta \mathrm{D} + \mathrm{i}k_x \sin\delta)^2 \times (\mathrm{D}^2 - k^2)]G(Z) = 0. \quad (13)$$

The corresponding boundary conditions are

$$G = (\mathrm{D}^2 - k^2)G = \mathrm{D}(\mathrm{D}^2 - k^2)G = 0 \quad \text{at} \quad Z = 0 \quad (14)$$

$$(\mathrm{D}^2 - k^2)G = 0, \quad \mathrm{D}G = -Bi\,G, \quad \mathrm{D}^2(\mathrm{D}^2 - k^2)G = Ma\,k^2 G \quad \text{at} \quad Z = 1 \quad (15)$$

where Bi and Ma are the Biot number and the Marangoni number, respectively

$$Bi \equiv \alpha L/\lambda \quad (16)$$

$$Ma \equiv (\sigma_t \Delta T L)/(\kappa\mu). \quad (17)$$

The Biot number represents the heat-transfer condition at the free surface where the heat-transfer coefficient is nondimensionalized by the thermal conductivity of the liquid and the depth of the layer. The Marangoni number represents the surface tension force relative to the viscous effect.

Onset of Marangoni convection under vertical magnetic field

Let us consider at first the case when the magnetic field is imposed in a vertical direction, namely $\delta = 0$ (Fig. 1). The effect of the inclined magnetic field will be discussed later.

In this case, equation (13) can be solved analytically under boundary conditions (14) and (15) and the critical Marangoni number Ma_c is obtained as

$$Ma_c = (M^2 C_3)/(C_1 \sinh k + C_2 \cosh k + C_3) \quad (18)$$

where

$$C_1 = \{(k \sinh k + Bi \cosh k)C_2 - (k^2 M)/(\sqrt{(M^2 + 4k^2)}(\cosh\sqrt{(M^2 + 4k^2)} - 1) + Bi\,C_3\}/(k \cosh k + Bi \sinh k)$$

$$C_2 = \beta \sinh\alpha - \alpha \sinh\beta$$

$$C_3 = (1/2)(\sqrt{(M^2 + 4k^2)} \sinh M - M \sinh\sqrt{(M^2 + 4k^2)})$$

$$\alpha = (1/2)\sqrt{(M + (M^2 + 4k^2))}$$

$$\beta = (1/2)(M - \sqrt{(M^2 + 4k^2)}).$$

When $M^2 \to 0$, equation (18) agrees with the solution obtained by Pearson [2] by asymptotic analysis.

Equation (18) represents the critical Marangoni number corresponding to a certain wave number. Therefore, the minimum value against the change of wave number represents the real critical Marangoni number and the corresponding wave number becomes the critical wave number. Hereafter, the minimum critical Marangoni number is referred to simply as the critical Marangoni number. The critical Marangoni number and the critical wave number are, respectively, denoted by Ma_c and k_c.

The critical Marangoni number and the critical wave number are listed in Table 1.

When $M^2 \to \infty$ and $Bi \to 0$, the critical wave number and the critical Marangoni number are expressed as the results of asymptotic analysis

$$k_c \to \{(1/2)M\}^{1/2} \quad (19)$$

$$(M^2 \to \infty, Bi \to 0)$$

$$Ma_c \to M^2. \quad (20)$$

The expressions for k_c and Ma_c for the case of $M^2 \to \infty$ and $Bi \to \infty$ are obtained in the same way

$$k_c \to (1/4)M \quad (21)$$

$$(M^2 \to \infty, Bi \to \infty)$$

$$Ma_c \to 8Bi\,M. \quad (22)$$

In the case of buoyancy convection, the critical Rayleigh number becomes independent of the Biot number at the free surface when M^2 is sufficiently large [5, 11].

On the other hand, the dependence of the critical Marangoni number on Bi is crucial as Nield [6] has pointed out, though the expressions obtained by Nield are slightly different from equations (20) and (22).

The critical Marangoni number, however, should become independent of Bi when M^2 is extremely large compared with Bi, which will be discussed below.

Figure 2 shows the dependence of the critical Marangoni number on the squared Hartmann number. The broken lines in the figure indicate the critical Marangoni number corresponding to zero magnetic field and the relations expressed by equations (20) and (22) are also indicated in the figure.

The effect of the magnetic field is negligibly small when the squared Hartmann number is smaller than unity. The effect becomes remarkable when $M^2 > 100$. The critical Marangoni number increases

Table 1. Critical Marangoni number and critical wave number in infinite liquid layer under vertical magnetic field

M^2	*Bi* 0	0.01	0.1	1.0	10.0	100.0	1000.0	10 000.0
0	Ma_c = 79.6067	79.9913	83.4267	116.127	413.440	3303.83	32 170.1	320 827
	k_c = 1.993	1.997	2.028	2.246	2.743	2.976	3.010	3.014
0.1	79.8645	80.2500	83.6933	116.467	414.409	3310.76	32 236.4	321 487
	1.995	1.999	2.030	2.249	2.746	2.980	3.014	3.018
1.0	82.1724	82.5657	86.0789	119.505	423.051	3372.42	32 826.7	327 363
	2.015	2.018	2.050	2.271	2.777	3.016	3.051	3.055
5.0	92.1834	92.6100	96.4202	132.616	459.903	3633.40	35 321.5	352 196
	2.094	2.098	2.132	2.364	2.903	3.165	3.204	3.208
10.0	104.223	104.688	108.844	148.260	503.007	3934.80	38 196.4	380 805
	2.181	2.185	2.220	2.465	3.042	3.331	3.375	3.380
20.0	127.111	127.647	132.431	177.691	581.952	4476.93	43 351.1	432 081
	2.325	2.329	2.367	2.634	3.278	3.617	3.670	3.676
50.0	189.873	190.586	196.954	256.912	784.055	5815.08	55 987.1	557 685
	2.630	2.635	2.680	2.995	3.800	4.271	4.352	4.360
100.0	284.222	285.177	293.686	373.432	1063.01	7568.02	72 362.6	720 264
	2.959	2.965	3.017	3.391	4.396	5.058	5.183	5.197
200.0	455.762	457.107	469.090	580.792	1527.05	10 302.8	97 529.2	969 686
	3.377	3.384	3.447	3.901	5.201	6.199	6.415	6.439
500.0	919.777	922.027	942.057	1127.31	2646.48	16 268.7	150 877	1.49654×10^6
	4.080	4.090	4.172	4.776	6.658	8.519	9.049	9.115
1000.0	1632.47	1635.91	1666.47	1947.33	4188.12	23 560.6	213 312	2.10960×10^6
	4.745	4.757	4.858	5.615	8.127	11.221	12.334	12.482
2000.0	2974.82	2980.17	3027.77	3462.53	6833.01	34 624.5	303 050	2.98388×10^6
	5.547	5.561	5.688	6.637	9.984	15.072	17.239	17.548
5000.0	6773.06	6782.92	6870.42	7663.45	13 581.9	58 779.7	483 849	4.72266×10^6
	6.863	6.882	7.050	8.325	13.170	22.292	26.950	27.678
10 000.0	12 830.2	12 846.0	12 986.5	14 252.8	23 444.8	89 249.6	693 281	6.70356×10^6
	8.092	8.116	8.324	9.909	16.256	29.676	34.988	35.931
20 000.0	24 562.5	24 588.1	24 815.5	26 855.6	41 280.8	137 798	943 258	—
	9.565	9.595	9.850	11.808	20.047	39.448	44.874	—
50 000.0	58 678.4	58 727.4	59 161.4	63 035.1	89 568.2	—	—	—
	11.963	12.001	12.335	14.904	26.359	—	—	—
100 000.0	114 213	114 293	115 005	121 338	163 789	—	—	—
	14.187	14.234	14.640	17.780	32.333	—	—	—

infinitely with the increase of M^2. It is clear that relation (20) is valid for $M^2 \to \infty$, $Bi \to 0$ and relation (22) also holds good for large M^2 and Bi, though relation (22) is a slight overestimation.

Although relations (20) and (22) cross at $M^2 = 64Bi^2$, the actual instability curves cannot cross each other. Relation (20) holds again when M^2 is extremely large compared with a given large Biot number, that is, $M^2 > 64Bi^2$. In other words, each of the curves approaches the relation $Ma_c \to M^2$ for extremely large M^2, even if the Biot number is large.

Figure 3 shows the dependence of the critical wave number on the squared Hartmann number. The effect of the magnetic field appears when $M^2 > 100$. The wave number increases infinitely with the increase of M^2. In other words, the distance between the cells becomes shorter as the intensity of the magnetic field increases.

Figure 4 shows the dependence of the critical Marangoni number on the Biot number. The critical Marangoni number increases in proportion to Bi when Bi is large. In the case of buoyancy convection [3, 5, 11] the critical Rayleigh number has a finite value even if the Biot number is infinite. On the contrary, the onset of Marangoni convection is completely suppressed when Bi is infinite.

Figure 5 shows the dependence of the critical wave number on the Biot number. The critical wave numbers approach constant values when the Biot number is either very small or very large and they change greatly in the intermediate region. The rate of change in that region becomes large with the increasing intensity of the magnetic field. The flow patterns become

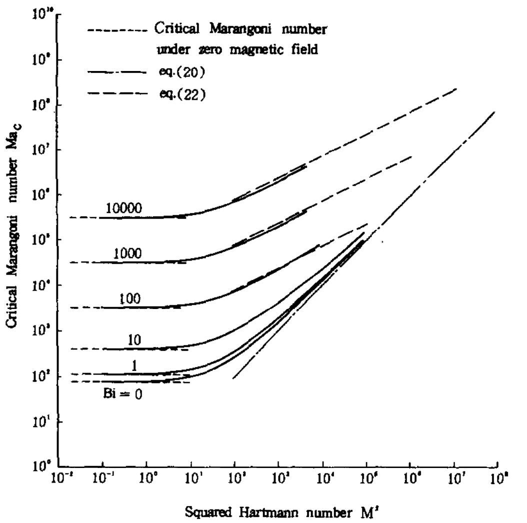

FIG. 2. Dependence of critical Marangoni number on squared Hartmann number.

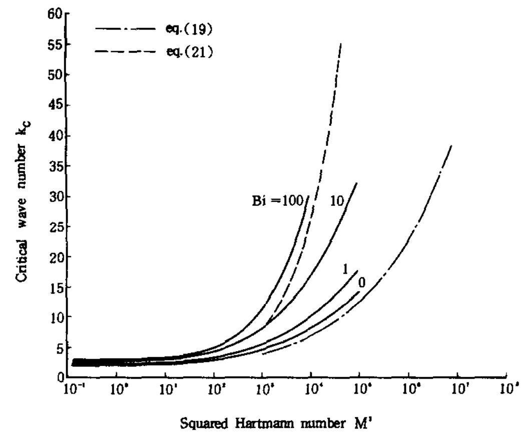

FIG. 3. Dependence of critical wave number on squared Hartmann number.

very sensitive to the Biot number as the Hartmann number increases.

Effect of orientation of magnetic field

When the magnetic field is inclined from the vertical, the new parameters M^* and C are used, which were introduced by Chandrasekhar [5]

$$M^* = M\cos\delta \tag{23}$$

$$C = k_x \tan\delta. \tag{24}$$

Equation (13) is rewritten as below by using M^* and C

$$[(D^2-k^2)^3 - M^{*2}(D+iC)^2(D^2-k^2)]G(Z) = 0. \tag{25}$$

The critical condition for the onset of Marangoni convection is expressed as follows by solving equation (25) under boundary conditions (14) and (15):

$$\det\begin{bmatrix} Mak^2 e^k & Mak^2 e^{-k} & [Mak^2-\alpha^2(\alpha^2-k^2)]e^{\alpha} & [Mak^2-\bar\alpha^2(\bar\alpha^2-k^2)]e^{-\bar\alpha} & [Mak^2-\beta^2(\beta^2-k^2)]e^{\beta} & [Mak^2-\bar\beta^2(\bar\beta^2-k^2)]e^{-\bar\beta} \\ (Bi+k)e^k & (Bi-k)e^{-k} & (Bi+\alpha)e^{\alpha} & (Bi-\bar\alpha)e^{-\bar\alpha} & (Bi+\beta)e^{\beta} & (Bi-\bar\beta)e^{-\bar\beta} \\ 0 & 0 & (\alpha^2-k^2)e^{\alpha} & (\bar\alpha^2-k^2)e^{-\bar\alpha} & (\beta^2-k^2)e^{\beta} & (\bar\beta^2-k^2)e^{-\bar\beta} \\ 0 & 0 & \alpha(\alpha^2-k^2) & -\bar\alpha(\bar\alpha^2-k^2) & \beta(\beta^2-k^2) & -\bar\beta(\bar\beta^2-k^2) \\ 0 & 0 & \alpha^2-k^2 & \bar\alpha^2-k^2 & \beta^2-k^2 & \bar\beta^2-k^2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} = 0 \tag{26}$$

where

$$\alpha = (1/2)[(M^*+\alpha_r)+i\alpha_i]$$
$$\beta = (1/2)[(M^*-\alpha_r)-i\alpha_i]$$
$$\alpha_r = (1/\sqrt{2})\sqrt{(\sqrt{((M^{*2}+4k^2)^2+16M^{*2}C^2)} + (M^{*2}+4k^2))}$$
$$\alpha_i = (1/\sqrt{2})\sqrt{(\sqrt{((M^{*2}+4k^2)^2+16M^{*2}C^2)} + (M^{*2}+4k^2))}$$

and the overbar $^-$ on the variable represents the conjugate complex number.

Table 2 shows the critical Marangoni number and the critical wave number for $M^{*2} = 10$ where C is changed from 0 to 10.

The critical Marangoni number corresponding to $C = 0$ is always the smallest even if M^{*2} and Bi are changed. This means that the rolls the axes of which are parallel to the horizontal component of the mag-

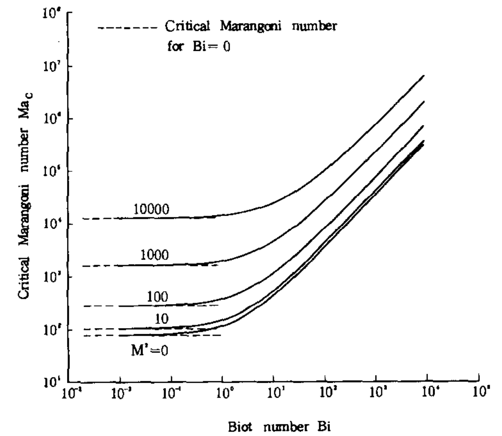

FIG. 4. Dependence of critical Marangoni number on Biot number.

Table 2. Critical Marangoni number and critical wave number in inclined magnetic field

C	Bi 0	1	10
0	$Ma_c = 104.2$ $k_c = 2.18$	148.3 2.46	503.0 3.04
0.5	104.6 2.18	148.8 2.47	504.3 3.06
1.0	105.8 2.18	150.3 2.48	508.4 3.07
5.0	140.4 2.43	194.0 2.75	620.0 3.46
10.0	223.5 2.88	295.2 3.32	851.9 4.25

netic field are generated when the magnetic field is inclined, while the Bénard-type cells are generated when the magnetic field is perpendicular to the horizontal liquid layer.

It is found that the effect of inclined magnetic field on the flow pattern of Marangoni convection is similar to that of the buoyancy convection analyzed by Chandrasekhar [5]. The horizontal component does not have any effect at all on the critical Marangoni number. What is effective in suppressing the onset of the Marangoni convection is the vertical component of the magnetic field, as in the case of buoyancy convection. The results shown in Figs. 2–5 are applicable for the case of an inclined magnetic field only if M^2 is replaced by M^{*2}

ONSET OF MARANGONI CONVECTION IN A CIRCULAR CYLINDRICAL CONTAINER

The onset of convective instability in a circular cylindrical container as shown in Fig. 6 is considered.

A vertical temperature gradient, decreasing from the bottom toward the top, and a vertical magnetic field are imposed on the liquid layer in the container, the side wall of which is thermally insulated.

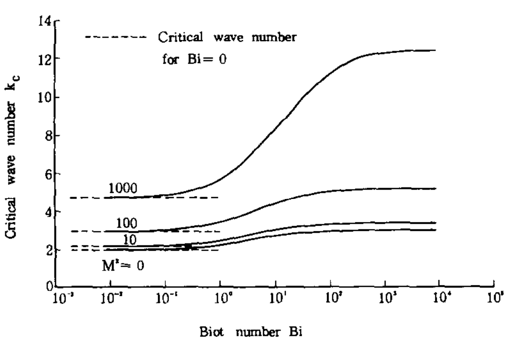

FIG. 5. Dependence of critical wave number on Biot number.

The aspect ratio A is defined as the ratio of the radius of the container to the depth of the liquid layer.

Analysis by Galerkin method

Let us assume that the steady convection occurs as two-dimensional concentric rolls at a marginal state.

Perturbation equations (3) and (8) should be expressed by a cylindrical coordinate system in this case.

Such a problem can be analyzed by the Galerkin method as Davis [8] and Catton [9] have done for the case of buoyancy convection.

V_z and θ are expanded, respectively, with a series of trial functions F_{ij} and G_{ij} which satisfy the corresponding boundary conditions

$$V_z = \alpha_{ij} F_{ij} \tag{27}$$

$$\theta = \beta_{ij} G_{ij} \tag{28}$$

where Einstein's convection of summation is applied and

$$F_{ij} = b_{0;i}(R/A) f_j(Z)$$

$$G_{ij} = J_0(\mu_i R/A) g_j(Z)$$

$$b_{n;m}(R) = \{J_n(\lambda_m R)\}/\{J_0(\lambda_m)\} - \{I_n(\lambda_m R)\}/\{I_0(\lambda_m)\}$$

$$f_j(Z) = (1-Z)Z^{j+1}$$

$$G_j(Z) = Z^j.$$

J_n and I_n are the Bessel function and the modified Bessel function of the first kind of order n, respectively.

λ_m and μ_m are the roots of the following equations:

$$b_{i;m}(1) = 0 \tag{29}$$

$$J_1(\mu_m) = 0. \tag{30}$$

The boundary conditions at the free surface, equations (31) and (32), as shown below are not satisfied

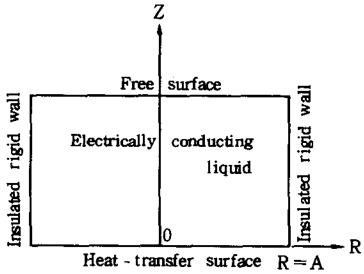

FIG. 6. Electrically conducting liquid in circular cylindrical container.

yet, but will be satisfied in the surface integrals

$$\Delta V_z = -Ma\,\Delta_{II}\theta \quad (31)$$

at $Z = 1$

$$\partial\theta/\partial Z = -Bi\,\theta \quad (32)$$

where Δ_{II} represents the two-dimensional Laplace operator related to a horizontal plane.

The following matrix equation is obtained by substituting equations (27) and (28) into equations (8) and (3) and applying the Galerkin method

$$\begin{pmatrix} \mathbf{A}_{11}-\mathbf{B}_{11}-M^2\mathbf{C}_{11} & -Ma\,\mathbf{A}_{12} \\ \mathbf{A}_{21} & -(\mathbf{A}_{22}+Bi\,\mathbf{B}_{22}) \end{pmatrix} \begin{pmatrix} \alpha_{ij} \\ \beta_{ij} \end{pmatrix} = 0 \quad (33)$$

where

$$\mathbf{A}_{11} = \int_v \Delta F_{mn}\Delta F_{ij}\,\mathrm{d}v$$

$$\mathbf{A}_{12} = \int_{Z=1} (\partial/\partial Z)F_{mn}\Delta_{II}G_{ij}\,\mathrm{d}S$$

$$\mathbf{A}_{21} = \int_v G_{mn}F_{ij}\,\mathrm{d}v$$

$$\mathbf{A}_{22} = \int_v \nabla G_{mn}\nabla G_{ij}\,\mathrm{d}v$$

$$\mathbf{B}_{11} = \int_{R=A} (\partial/\partial R)F_{mn}\Delta F_{ij}\,\mathrm{d}S$$

$$\mathbf{B}_{22} = \int_{Z=1} G_{mm}G_{ij}\,\mathrm{d}s$$

$$\mathbf{C}_{11} = \int_v F_{mn}(\partial^2/\partial Z^2)F_{ij}\,\mathrm{d}v$$

where the integrals with dS and dv are the surface and the volumetric integrals, respectively.

Marangoni convection occurs only when the coefficients α_{ij} and β_{ij} have non-trivial solutions. The condition is given below

$$\det[(1/Ma)I-(\mathbf{A}_{11}-\mathbf{B}_{11}-M^2\mathbf{C}_{11})^{-1} \times \mathbf{A}_{12}(\mathbf{A}_{22}+Bi\,\mathbf{B}_{22})^{-1}\mathbf{A}_{21}] = 0 \quad (34)$$

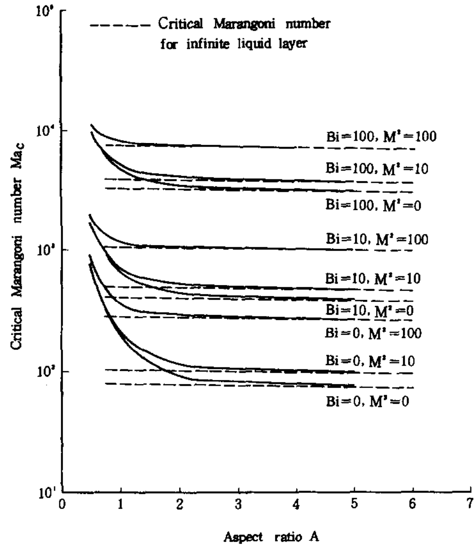

FIG. 7. Dependence of critical Marangoni number on aspect ratio.

where I denotes the unit matrix and the quantities with exponent -1 represents the inverse matrix.

Equation (34) is an eigenvalue equation where $1/Ma$ is the eigenvalue. The Marangoni number corresponding to the maximum eigenvalue for given M^2, Bi and A represents the critical Marangoni number.

RESULTS AND DISCUSSION

Critical Marangoni numbers are listed in Table 3.

The dependence of the critical Marangoni number on the aspect ratio is shown in Fig. 7 for the cases of $Bi = 0$, 10 and 100, where broken lines indicate the critical Marangoni number for the infinite liquid layer. As is expected, the critical Marangoni number increases as the aspect ratio decreases since the disturbances are damped down in the vicinity of the side

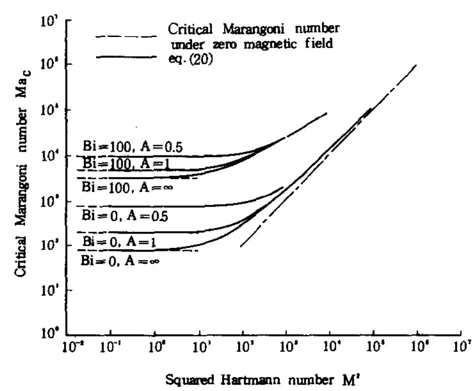

FIG. 8. Dependence of critical Marangoni number on squared Hartmann number.

Table 3. Critical Marangoni number in circular cylindrical container

M^2	A 0.5	1.0	3.0	5.0
		(a) $Bi = 0$		
0	774	204.3	86.38	81.85
1	776	206.0	88.97	84.40
10	789	221.1	110.9	106.4
100	920	367.6	291.8	286.8
1000	2110	1685	1653	1640
		(b) $Bi = 1$		
0	869	253.5	124.0	81.85
1	870	255.6	127.4	84.40
10	885	274.3	155.8	106.4
100	1032	455.3	381.5	286.8
1000	2364	2030	—	—
		(c) $Bi = 10$		
0	1694	677.8	431.6	420.0
1	1697	683.3	441.1	429.6
10	1726	732.4	521.4	509.5
100	2010	1198	1079	1074
1000	4550	4250	4240	—
		(d) $Bi = 100$		
0	9642	4798	3421	3345
1	9660	4836	3489	3413
10	9820	5174	4045	3976
100	11400	8260	7644	—
1000	24200	23380	—	—

wall because the boundary condition

$$V_R = V_Z = \partial\theta/\partial R = 0.$$

However, the effect of the aspect ratio on the critical Marangoni number becomes smaller with the increase of the Hartmann number and the Biot number.

Figure 8 shows the dependence of the critical Marangoni number on the squared Hartmann number where broken lines indicate the critical Marangoni number under zero magnetic field.

As mentioned previously, with the increasing intensity of magnetic field, the critical Marangoni number becomes independent of the aspect ratio and approaches the value in the infinite liquid layer.

The distribution of the vertical component of velocity on the horizontal plane $Z = 1/2$ is illustrated in Fig. 9 for $Bi = 0$ where the velocity on the axis $R = 0$ is normalized as 1. The velocity distribution in the infinite liquid layer is also indicated for comparison which is expressed below in the case of the cylindrical coordinate system

$$V_Z = J_0(k_c R) \quad (A \to \infty) \tag{35}$$

where k_c is the critical wave number in the infinite liquid layer which has been obtained in the previous sections.

The velocity component for $A = 1$ is suppressed to zero because of the effect of the side wall and it differs remarkably from those for $A = 3, 5$ and ∞ when the Hartmann number is zero (Fig. 9(a)).

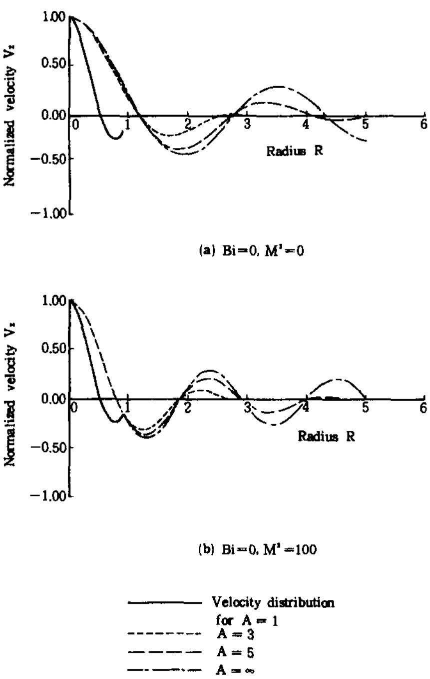

FIG. 9. Velocity distribution in container.

The difference, however, becomes small with the increase of the Hartmann number and the Biot number (Fig. 9(b)), which explains why the effect of the aspect ratio vanishes and the critical Marangoni number approaches that of the infinite liquid layer when M^2 is large. The distance between the rolls becomes shorter and the velocity and temperature fields are unaffected by the existence of the side wall when M^2 and Bi are large.

CONCLUSION

The onset of Marangoni convection in the horizontal layer of an electrically conducting liquid has been studied theoretically and the following results have been obtained.

(1) The effect of the magnetic field on the onset of Marangoni convection is negligibly small when the squared Hartmann number M^2 is smaller than unity.

(2) The critical Marangoni number Ma_c for large M^2 is expressed by the following relations:

for small Bi, $Ma_c \to M^2$;
for large Bi, $Ma_c \to 8BiM$ (for $M^2 \lesssim 64Bi^2$)
$Ma_c \to M^2$ (for $M^2 \gtrsim 64Bi^2$).

(3) The critical Marangoni number increases in proportion to Bi when Bi is large.

(4) The rolls, the axes of which are parallel to the horizontal component of the magnetic field, are generated in the case when the magnetic field is inclined.

(5) The effect of the aspect ratio of the liquid layer

on both the critical Marangoni number and the velocity field vanishes at large M^2.

REFERENCES

1. L. G. Napolitano, Marangoni convection in space microgravity environments, *Science* **225**, 197–198 (1984).
2. J. R. A. Pearson, On convective cells induced by surface tension, *J. Fluid Mech.* **4**, 489–500 (1958).
3. D. A. Nield, Surface tension and buoyancy effect on cellular convection, *J. Fluid Mech.* **19**, 341–352 (1964).
4. L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media: Course of Theoretical Physics*, Vol. 8. Pergamon Press, London (1966).
5. S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability*. Clarendon Press, London (1961).
6. D. A. Nield, Surface tension and buoyancy effect in the cellular convection of an electrically conducting liquid in a magnetic field, *Z. Angew. Math. Phys.* **17**, 131–139 (1966).
7. N. Rudraiah, V. Ramachandramurthy and O. P. Chandna, Effects of magnetic field and non-uniform temperature gradient on Marangoni convection, *Int. J. Heat Mass Transfer* **28**, 1621–1624 (1985).
8. S. H. Davis, Convection in a box; linear theory, *J. Fluid Mech.* **30**, 465–478 (1967).
9. I. Catton, Convection in a closed rectangular region: the onset of motion, *J. Heat Transfer* **92**, 186–187 (1970).
10. J. S. Vrentas, R. Narayanan and S. S. Agrawal, Free surface convection in a bounded cylindrical geometry, *Int. J. Heat Mass Transfer* **24**, 1513–1529 (1981).
11. T. Maekawa and I. Tanasawa, Effects of magnetic field and buoyancy on onset of Marangoni convection, *Appl. Microgravity Technol.* (1988), in press.

EFFET DU CHAMP MAGNETIQUE SUR L'APPARITION DE LA CONVECTION DE MARANGONI

Résumé—On étudie théoriquement l'apparition de la convection de Marangoni dans une couche horizontale de liquide conducteur d'électricité, auquel on applique un gradient vertical de température et un champ magnétique. La solution analytique est obtenue pour la condition critique de l'apparition de la convection de Marangoni dans une couche infinie de liquide et la solution numérique pour une couche finie liquide confinée dans un conteneur cylindrique. On clarifie les effets du champ magnétique, du nombre de Biot à la surface libre et le rapport de forme de la couche liquide. On obtient aussi le comportement asymptotique du nombre critique de Marangoni pour une grande valeur du nombre de Hartmann. On trouve que le nombre de Marangoni critique et le nombre de rouleaux qui se génèrent dans un état marginal augmentent tous deux avec l'intensité du champ magnétique, et que l'effet du rapport de forme de la couche liquide, du nombre de Marangoni critique et des champs de vitesse et de température, devient faible quand le champ magnétique est intensifié. Il est clair que les rouleaux sont générés quand le champ magnétique est incliné, tandis que les cellules de type Bénard sont générées avec un champ magnétique vertical dans le cas d'une couche liquide infinie.

DER EINFLUSS EINES MAGNETISCHEN FELDES AUF DAS EINSETZEN DER MARANGONI-KONVEKTION

Zusammenfassung—Das Einsetzen der Marangoni-Konvektion in einer horizontalen Schicht einer elektrisch leitenden Flüssigkeit wird theoretisch untersucht. Dabei wird dieser Schicht ein vertikaler Temperaturgradient und ein magnetisches Feld aufgeprägt. Die analytische Lösung ergibt die kritischen Bedingungen für das Einsetzen der Marangoni-Konvektion in einer unendlichen Flüssigkeitsschicht. Eine numerische Analyse wurde für eine endlich ausgedehnte Flüssigkeitsschicht in einem kreiszylindrischen Behälter durchgeführt. Die Einflüsse des magnetischen Feldes, der Biot-Zahl an der freien Oberfläche und des Längenverhältnisses der Flüssigkeitsschicht sind deutlich geworden. Das asymptotische Verhalten der kritischen Marangoni-Zahl bei großer Hartmann-Zahl wurde auch ermittelt. Es zeigt sich, daß die kritische Marangoni-Zahl und die Zahl der Konvektionszellen, die bei einem Grenzzustand entstehen, mit der Intensität des Magnetfeldes zunehmen. Der Einfluß des Längenverhältnisses der Flüssigkeitsschicht auf die kritische Marangoni-Zahl und das Geschwindigkeits- und Temperaturfeld wird mit stärker werdendem magnetischen Feld kleiner. Man erkennt auch, daß bei geneigtem Magnetfeld Konvektionszellen erzeugt werden, während Bénard-Zellen bei vertikalem Magnetfeld im Falle einer unendlichen Flüssigkeitsschicht entstehen.

ВЛИЯНИЕ МАГНИТНОГО ПОЛЯ НА ВОЗНИКНОВЕНИЕ КОНВЕКЦИИ МАРАНГОНИ

Аннотация—Теоретически исследуется возникновение конвекции Марангони в горизонтальном слое электропроводной жидкости в случае вертикального температурного градиента и приложения магнитного поля. Получено аналитическое решение для критических условий возникновения конвекции Марангони в бесконечном слое жидкости, проведен численный анализ для цилиндра. Исследовано влияние магнитного поля, числа Био на свободной поверхности и отношения сторон слоя жидкости. Найдено асимптотическое поведение критического числа Марангони для больших чисел Гартмана. Обнаружено, что критическое число Марангони и количество ячеек в виде вала, возникающих в промежуточном состоянии, увеличиваются с ростом напряженности магнитного поля, а влияние отношения сторон слоя на критическое число Марангони и поле скорости и температуры уменьшается при увеличении напряженности магнитного поля. Найдено, что в случае бесконечного слоя жидкости ячейки в виде вала образуются при наклонном магнитном поле, а ячейки Бенара—при вертикальном.